Report on Certain Aspects of UPS Proposal One

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Introduction

United Parcel Service (UPS) has proposed an *ad hoc* method for attributing seasonal costs to competitive products, either as a group or to individual products.[1] The justification for its proposal rests on the assertion that seasonal costs are caused solely by competitive products and should thus be included in the incremental cost for those products.[2] A review of the relative volumes in December makes it obvious that this assertion is mistaken, however, as First-Class Mail contributes to the seasonal peak, along with the various competitive products.

The existence of more than one product experiencing seasonal volume increases raises the question of how any additional seasonal costs should be included in the incremental costs of the relevant products. The importance of avoiding broad assumptions in the attribution process, and properly identifying the causal linkages, has been emphasized by the Commission in its analysis of the proper attribution of seasonal annex costs:[3]

> The Postal Service contends that "[t]here are multiple reasons that could lead to space shortages during peak season, but one of the primary reasons is the increase in the volume of parcel-shaped mail." 2019 FSUS Report at 24. For this reason, it adds the peak adjustment annex space increases to the "MODS MANP," "MODS ISC," "NDC MANP," and "NONMODS MANP" space category totals. This

[1] See, Petition of United Parcel Service, Inc. For the Initiation of Proceedings to Make Changes to Postal Service Costing Methodologies, (UPS Petition), Docket No. RM2020-9, May 29, 2020, at 20.

[2] Id. at 24.

[3] See, Postal Regulatory Commission, Order 5637, Docket No. RM2020-1, Aug. 17, 2020, at 20.

> explanation is feasible for a number of the peak annex adjustments listed in the "FACILITY SPACE SUMMARY.xlsx" file. However, the Commission asks the Postal Service to verify in its next ACR filing the assumption (particularly for those annex spaces leased 4 months and longer) that the annex space usage is entirely due to packages and that the supplemental space should only be added to the space category totals identified in the Proposal Nine methodology. (Footnotes omitted).

With further investigation, the Postal Service may find that peak annex costs are indeed caused solely by parcels. But regardless of the outcome of the Postal Service's verification, the key point is that the causal link has been carefully examined and the attribution is not based upon a broad assumption.

In this report, I describe the appropriate way to incorporate seasonal costs into products' incremental costs, when more than one product experiences a volume increase during the peak. The appropriate method should be consistent with both established costing theory and the Postal Service's actual operational practice. I then review the application of the appropriate method to several cost components. The report concludes with an analysis of the *ad hoc* method proposed by UPS for calculating the amount of seasonal cost to be attributed to competitive products.

Operational Responses to Peak Season Volumes

The appropriate treatment of seasonal cost depends, in part, on the operational response to peak volumes. The Postal Service's response starts months in advance of the peak, as the Postal Service plans its resource use for that period. While the exact sizes and timings of the different volume peaks vary from year to year, much of the peak is predictable, and the Postal Service plans its resource deployment around the

expected seasonal volumes. It must allow for contingencies, however, because conditions are not perfectly predictable. For example, although weather is not the primary driver of additional cost during the December peak, it is an important contributor to higher operating costs. Bad weather makes both transportation and delivery more expensive during the December peak.

Analysis of the Postal Service's operational response to seasonal volume changes reveals that those responses occur in three different ways. First, the Postal service uses existing resources and methods more intensively to handle the peak volume. Examples are increasing capacity utilization on highway transportation and higher productivity in city carrier street time. Second, the Postal Service uses additional amounts of existing resources and methods of handling peak volume. Examples are acquisition of additional air transportation capacity and addition of supplemental mail processing labor. Third, the Postal Service adds different resources and/or uses different methods to handle the peak volume. Examples are using SPR hours to deliver packages in high-volume areas and leasing annexes for handling peak volumes.

Incorporating Seasonal Costs into Attributable Costs

Attributable costs are annual measures, not monthly measures. The costs reported in the Annual Compliance Report are for the whole year and include both peak and off-peak costs. Accurate product costing requires that these two types of costs are calculated in a consistent way, so they can be combined into a meaningful, accurate, single annual cost measure. Moreover, there is nothing inherent in the nature of

seasonal costs that would exclude them from being calculated using the well-established methodology for attributing any type of cost to products.

The relevant variabilities and distribution keys may be different for seasonal costs, but the established volume variable cost methodology is directly applicable. The same is true for the established incremental cost methodology.

The nature of the Postal Service's response determines how the calculation of seasonal volume variable and incremental costs should be pursued. If the Postal Service responds to peak volume by using existing resources more intensively, then the cost response is similar to what happens the rest of the year. In this case, volume variable and incremental costs would be captured within the existing product costing models, and existing variabilities and distribution keys would be applicable.

If the Postal Service response is to add additional amounts of existing resources using current methods, then the need for a different model and parameters depends upon that amount and effect of the additional resources used. If productivity is affected in a material way by the application of additional resources, or if the seasonal resource use is far beyond usage during the rest of the year, application of a separate model may be justified. The determination should ultimately be an empirical one.

Finally, if the Postal Service uses new resources, or different methods, then a new costing analysis is justified to examine the cost causality embodied in the new methods and resources. This implies estimation of a separate model and possibly the construction of a separate distribution key. But even in this case, with different parameters, the established methodology is appropriate and necessary. The importance of identifying and measuring the causal linkage between volume and cost is just as

essential for seasonal costs as it for non-seasonal costs. Application of an *ad hoc* model, like UPS's Proposal One, will likely lead to errors in attributing costs to products. In fact, Proposal One produces erroneous incremental costs for competitive products.

This can be seen with a simple quadratic model with two products, A and B, and a single seasonal peak. To ensure clarification of the role of the established methodology, we will assume that operations are different during the seasonal peak, so that a separate model and parameters are applicable during that period. Let an "A" subscript represent the first product (which could be the group of competitive products) and a "B" subscript represent the second product (which could be First Class Mail). The "N" subscript refers to non-peak periods and the" P" subscript refers to the peak period. The β coefficients represent non-peak cost parameters and the δ coefficients represent peak cost parameters. Finally, β_0 and δ_0 represent common costs during the non-peak and peak periods and β_{0A}, β_{0B}, δ_{0A} and δ_{0B} are the product specific costs in the two periods.

With these definitions in place, total cost, the sum of peak and non-peak cost, is given by:

$$C = \beta_0 + \beta_{0A} + \beta_{0B} + \beta_{1A}V_{AN} - \beta_{2A}V_{AN}^2 + \beta_{1B}V_{BN} - \beta_{2B}V_{BN}^2 - \beta_3 V_{AN}V_{BN} + \delta_0 + \delta_{0A} + \delta_{0B} + \delta_{1A}V_{AP} - \delta_{2A}V_{AP}^2 + \delta_{1B}V_{BP} - \delta_{2B}V_{BP}^2 - \delta_3 V_{AP}V_{BP}.$$

Incremental cost for Product A is the difference between total cost and the cost remaining when Product A is removed from the production vector:[4]

$$IC_A = \beta_{0A} + \beta_{1A}V_{AN} - \beta_{2A}V_{AN}^2 - \beta_3 V_{AN}V_{BN} + \delta_{0A} + \delta_{1A}V_{AP} - \delta_{2A}V_{AP}^2 - \delta_3 V_{AP}V_{BP}.$$

The incremental cost for Product A includes volume variable, product specific and infra-marginal costs from both the peak and non-peak periods, and does so in an internally consistent manner. In contrast, the proposed UPS Proposal One approach is to calculate the incremental cost of product A by combining all non-volume variable cost from the peak season with the incremental cost for the non-peak season. This proposal leads to several problems, including an inconsistent incremental cost measure, which can be identified by calculating the non-volume variable peak costs, $NVVC_P$ to be applied. Non-volume-variable peak cost is found by subtracting total volume variable peak costs for Products A and B from total peak costs. That subtraction yields:[5]

$$NVVC_P = \delta_0 + \delta_{0A} + \delta_{0B} + \delta_{2A}V_{AP}^2 + \delta_{2B}V_{BP}^2 + \delta_3 V_{AP}V_{BP}.$$

[4] The derivations of incremental cost, marginal cost, volume variable cost, non-volume-variable cost, and a Proposal One type of incremental cost are all presented in the mathematical appendix to this report.

[5] This derivation is provided in the mathematical appendix. Note that the second-order and cross-product terms enter the expression for non-volume-variable cost with a positive coefficient because they enter total peak costs with a coefficient of minus one, but enter volume variable cost with a coefficient of minus two. The larger coefficient in volume variable costs arises from taking the derivative of cost with respect to volume squared.

Inspection of the non-volume-variable peak costs reveals that it includes terms that are not specific to Product A, meaning these costs would remain even if Product A were to be removed. Such costs should not be included in Product A's incremental cost, and doing so will result in overstating Product A's true incremental cost.

Moreover, Proposal One is silent on what else should be included in the incremental cost for the group of competitive products. Is UPS proposing that the non-volume-variable seasonal costs be added to the incremental cost of competitive products as they are currently calculated? If so, Proposal One is specifying double-counting, as some of the seasonal non-volume-variable cost is already included in competitive products' incremental cost. To avoid double counting, one could propose adding the non-volume-variable seasonal cost to just the volume-variable cost for competitive products. But even this approach does not provide the correct incremental cost measure. Calculating incremental costs for this version of Proposal One ($\widetilde{IC}_A$) reveals a measure that incorporates illegitimate terms involving the costs for Product B and omits relevant terms like the peak period economies of scope measure:

$$\widetilde{IC}_A = \delta_0 + \delta_{0A} + \delta_{0B} + \delta_{1A}V_{AP} - \delta_{2A}V_{AP}^2 + \delta_{2B}V_{BP}^2 + \beta_{1A}V_{AN} - 2\,\beta_{2A}V_{AN}^2 - \beta_3 V_{BN}V_{AN}$$

In addition, including costs related to Product B in the measure of Product A's incremental cost raises a dilemma about how to calculate Product B's incremental cost. If, for example, δ_{0B} and $\delta_{2B}V_{BP}^2$ are excluded from Product B's incremental cost, it will be understated, but if they are included, then double counting will occur. Proposal One is

not complete in describing its proposed method of calculating incremental costs for competitive and other products.

The existence of higher or lower volumes and costs in certain months does not, by itself, justify the inclusion of those costs in the incremental cost for individual products. Proper application of the "but for" approach to calculating incremental costs requires careful analysis to ensure there are reliably established underlying causal links between the individual products and the seasonal costs. This is particularly important in a network setting, in which throughput capacity is often jointly determined. The appropriate way to calculate seasonal incremental costs is to integrate that calculation with the calculation of non-seasonal incremental costs. To not do so can lead to erroneous calculation formulae, as in the UPS proposal.

Postal Service Applications of the Established Methodology to Seasonal Costs

In its petition, UPS asserts that the Postal Service costing models ignore peak season costs.[6] This assertion is in error, as it misses the various efforts the Postal Service has made to explicitly address and attribute peak season costs. These efforts have touched on the major cost segments such as delivery, transportation and mail processing.

First, contrary to the assertion by UPS, the additional costs associated with any seasonal peak volume are included in the accrued cost base from which attributable costs are derived. That is, the established models include seasonal costs when

[6] See, UPS Petition, Docket No. RM2020-9, May 29, 2020, at 3.

calculating volume variable and incremental costs. By ignoring this fact, the UPS petition dramatically understates the degree to which seasonal costs are attributed to products and dramatically overstates what it calls the "unexplained" increase in December costs.

Second, different parts of the Postal Service network will be impacted in different ways by seasonal fluctuations in volume. A large seasonal response in one part of the network may reduce the need for a seasonal response in another part. For example, an expansion of SPR hours to assist in delivering parcels during the seasonal peak reduces the need for additional hours on regular letter routes. This variation suggests that a single broad-brush approach to analyzing peak costs, like in Proposal One, will be inaccurate. Rather, it is important to individually consider each potential seasonal cost, reliably identify the nature of cost causality, and accurately link those costs to products either within the existing product cost structure or through an addition to or refinement of that structure.

For example, in the area of city carriers, the Postal Service recently completed, and the Commission approved, a study of special purpose route (SPR) carriers.[7] The Postal Service collected data specifically from the peak season and estimated separate variability models for that time of year. It explicitly investigated and accounted for differences in the activities by SPR carriers during the peak season and accounted for the change in volume mix. The study also estimated separate models for SPR carrier hours which arise from the need to assist regular letter carriers deliver packages.

[7] See, Docket No. RM2019-6, Proposal One, June 21, 2019, at 3.

Furthermore, the Postal Service is currently in the process of investigating the possibility of estimating an alternative seasonal street time model for regular letter carriers based upon a city-carrier data set that spans a full year.

As part of its update and refinement of the way building space costs are attributed to products, the Postal Service separately identified and attributed the cost of seasonal annexes which are rented during the peak season to assist in the handling of the volume peak.[8] The Postal Service's initial approach is to assume the annexes are used for handling peak-season parcels and attributed the cost to those products.

In the area of purchased highway transportation, the Postal Service completed a study of seasonal highway transportation costs.[9] The Postal Service noted the growth in costs for what are known as Christmas accounts and estimated separate variability equations for them, leading to materially higher variabilities than the proxies that were in place. It also investigated and estimated separate models for Dynamic Route Optimization transportation contracts which are also used during the peak season. Furthermore, with the increase in use of Christmas transportation contracts, the Postal Service is investigating the feasibility of separately sampling those contracts in TRACS, raising the possibility of constructing a separate distribution key for that transportation.

Finally, in a recent study of mail processing costs, the Postal Service collected data for the peak season and empirically investigated whether different variabilities should be applied for that period of the year.[10]

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[8] See, Docket No. RM2020-1, Order No. 5637, August 17, 2020, at 9.

[9] See, Docket No. RM2021-1, Proposal Seven, November 9, 2020 at 2-3.

[10] See, Docket No. RM2020-13, Proposal Six, September 15, 2020 at 5.

The UPS Proposed Approach Does Not Embody the Established Costing Methodology and Has Significant Drawbacks.

In contrast to the careful, function-by-function analysis of peak season costs in the established methodology, UPS proposes an *ad hoc*, broad-brush approach that would be applied, in the same way, to all cost segments.

UPS calculates what it calls "unexplained" costs in a multi-step process that it asserts represents the costing models that comprise the established methodology.[11] The UPS calculation is done separately for city and rural carriers, purchased transportation, clerks, and "other," which is made up of the remaining cost segments, excluding a small number of components from cost segment 18. For each type of cost, the UPS method first finds the annual unit costs for groups of products, such as First-Class Mail, Marketing Mail, Periodicals or all competitive products.

These unit costs are calculated by taking the FY 2019 Cost Segment volume variable cost for the product grouping and dividing that by the sum of the product grouping's FY2019 monthly volumes from the Postal Service's monthly unaudited financial information. More formally, the unit cost for product grouping "j" in cost segment "i" is given by:

$$UC_{ij} = \frac{VVC_{ij}}{\sum_{t=1}^{12} V_{jt}},$$

where UC_{ij} is the unit cost, VVC_{ij} is the FY 2019 segment volume variable cost for product group "j", and each V_{jt} is an FY 2019 monthly volume for product grouping "j." These calculated unit costs are then used to find the predicted change in monthly

[11] See, UPS Petition, Docket No. RM2020-9, May 29, 2020, at 20-25.

volume variable cost, by product. The predicted changes in costs ($\widehat{\Delta \mathrm{VVC}_{ijt}}$) are found by multiplying each product grouping's unit cost by the change in monthly volume:

$$\widehat{\Delta \mathrm{VVC}_{ijt}} = UC_{ij} * \Delta \mathrm{V}_{jt}.$$

The predicted total change in each segment's cost is the sum of the predicted change in costs for the various product groupings:

$$\widehat{\Delta \mathrm{VVC}_{it}} = \sum_{j=1}^{n} \widehat{\Delta \mathrm{VVC}_{ijt}}.$$

UPS then defines its "unexplained" cost change ($\widehat{\Delta \mathrm{UXC}}$) as the difference between the accrued cost change for the segment (ΔC_{it}) and its predicted cost change:

$$\widehat{\Delta \mathrm{UXC}_{it}} = \Delta C_{it} - \widehat{\Delta \mathrm{VVC}_{it}}.$$

In reality, the UPS method is just predicting volume variable cost for each month. This can be seen by using the definitions of the changes in cost and volume:

$$\widehat{\Delta \mathrm{VVC}_{it}} = \widehat{VVC}_{it} - \widehat{VVC}_{it-1}\,; \quad \Delta \mathrm{V}_{jt} = V_{jt} - V_{jt-1.}$$

Substitution yields:

$$\widehat{VVC}_{it} = \widehat{VVC}_{it-1} + \sum_{j=1}^{n} UC_{ij} * \left(V_{jt} - V_{jt-1}\right).$$

$$\widehat{VVC}_{it} = \widehat{VVC}_{it-1} + \sum_{j=1}^{n} UC_{ij} V_{jt} - \sum_{j=1}^{n} UC_{ij} V_{jt-1}$$

$$\widehat{VVC}_{it} = \sum_{j=1}^{n} UC_{ij} V_{jt}.$$

The last equality is important because it demonstrates the UPS calculates monthly volume variable cost by multiplying a fixed annual unit cost by each month's associated volumes and claims this represents the method used in the established methodology. But it does not. The established methodology works the other way. It takes the annual or monthly accrued cost for the cost segment (C_{it}) and multiplies it by the relevant variability (ε_i).[12]

$$VVC_{it} = \varepsilon_i C_{it.}$$

While this might seem like a somewhat esoteric difference in theory, it has an important implication in reality. The established methodology accounts for the fact that accrued costs increase in heavy volume months; the UPS method does not. This difference causes the UPS method to materially understate the volume variable cost for the various functions. For example, as shown in Table 1, the UPS method predicts a $21.3 million decline in combined city and rural volume variable costs for December 2018, whereas the CRA would project a $121 million increase. [13]

[12] In reality, this calculation should be done at the cost component level, just as the calculation of unit volume variable cost is done at the level. To be consistent with the UPS exercise, I also calculated the volume variable cost at the cost segment level.

[13] The program that produces the predicted cost changes is presented in Folder USPS-RM2020-9-1.

Table 1

UPS Predicted and Established Volume Variable Cost Changes from November to December FY 2019

Cost Segment	Trial Balance Cost Change	UPS Predicted VV Cost Change	Established VV Cost Change
Clerks	$185,859,827	$89,461,116	$152,477,934
City Carriers	$155,920,327	-$19,155,491	$74,897,727
Rural Carriers	$128,770,154	-$2,124,613	$46,200,552
Transportation	$221,026,059	$116,506,245	$174,657,572
Sum	$691,576,367	$184,687,257	$448,233,785

Understating volume variable costs for December necessarily implies overstating the "unexplained" costs as they arise, in the UPS methodology, as the difference between the recorded trial balance costs and predicted volume variable costs. As Table 2 shows, the UPS method produces "unexplained" costs that are often two to three times as large those calculated within the established methodology.[14]

[14] The program that calculates the "unexplained" cost changes is presented in Folder USPS-RM2020-9-1

Table 2
UPS Predicted and Established "Unexplained" Changes Cost for December FY 2019

Cost Segment	UPS "Unexplained" Cost Change	Established "Unexplained" Cost Change
Clerks	$96,398,711	$33,381,893
City Carriers	$175,075,817	$81,022,600
Rural Carriers	$130,894,767	$82,569,601
Transportation	$104,519,813	$46,368,487
Sum	$506,889,108	$243,342,581

One other aspect of the proposed UPS method bear mention. The UPS proposed method leads to an "unexplained" cost change of -$129 million in the "other category." UPS is silent on what negative "unexplained" costs mean and how these costs could fall during the seasonal volume peak.

UPS states that its calculated costs are unexplained or unaccounted for by the established cost models. But this assertion rests upon an assumption that these costs have not been studied and are not understood. This is not accurate. The costs calculated in the proposed UPS model are an estimate of the institutional costs for each function, for the month being studied.[15] Institutional costs are a well-established costing concept and include the costs associated with constructing and maintaining the Postal

[15] Another indication that the UPS method is erroneous is the fact that the sum of its 12 monthly institutional costs do not equal the annual amount of institutional costs in the established model. For example, for cost segment 14, the UPS method calculates 1.753 billion in institutional costs, but the actual amount is 1.717 billion.

Service's various networks. They also include activities associated with the provision of all, or broad groupings of products that are not caused by individual products. Finally, they included infra-marginal costs which arise from economies of density and scale in the different Postal functions.

The UPS petition focuses solely on the volume change in December and thus does not address the fact that volume changes occur in other months of the year. If it is appropriate to attribute "unexplained" cost change in one month to the volume changes that take place in that month, then it would seem to be appropriate to employ the same method for other months of the year.

For example, Table 3 presents the trial balance cost changes, along with the UPS projected volume variable cost change and its associated "unexplained" cost change for January 2019.[16] The table shows that there are large negative values for the "unexplained" cost changes. At the same time, competitive products' volume fell dramatically in January 2019, by 172.3 million pieces, a decline which was larger than the December increase. According to the UPS costing method, these cost reductions should be thus included in the incremental cost for competitive products. Doing so substantially offsets the increases in cost that occurred in December 2018. Perhaps these results suggest that focusing on the level of costs, when calculating incremental cost, is more appropriate than relying on changes in costs.

[16] The program that produces the predicted cost changes is presented in Folder USPS-RM2020-9-1.

Table 3
UPS Predicted and "Unexplained" Cost Changes for January 2019

Cost Segment	Trial Balance Cost Change	UPS Predicted VV Cost Change	UPS "Unexplained" Cost Change
Clerks	-$183,160,497	-$128,833,476	-$54,327,021
City Carriers	-$177,985,760	-$23,453,600	-$154,532,161
Rural Carriers	-$101,310,986	-$23,313,045	-$77,997,942
Transportation	-$206,561,453	-$142,718,459	-$63,842,994

Another example of applying the methodology to another month is given by April 2019. April is a month that experienced an increase in competitive product's volume but, as Table 4 demonstrates, a negative "unexplained" overall cost change.[17] UPS does not explain the implications of its proposed method for cost attribution for competitive products when their volumes rise but the "unexplained" costs fall.[18]

[17] Id.

[18] The UPS method predicted a decline in transportation costs despite an overall increase in volume because of declines in the volumes for Package Services and International Mail. Although these product groupings have relatively small volumes, they have high unit transportation costs under the UPS method. Thus, their volume declines lead to an overall predicted decline in volume variable costs when total volume is rising.

Table 4
UPS Predicted and "Unexplained:" Cost Changes for April 2019

Cost Segment	Trial Balance Cost Change	UPS Predicted VV Cost Change	UPS "Unexplained" Cost Change
Clerks	-$15,427,089	$6,817,652	-$22,244,742
City Carriers	-$28,433,779	$9,146,715	-$37,508,493
Rural Carriers	-$833,905	$3,019,563	-$3,853,469
Transportation	$4,086,350	-$1,562,624	$5,648,974

Mathematical Appendix

This appendix presents the derivations for the cost model presented in the report. The model includes volume for two different products, Product A and Product B. There are two time periods, a peak period (P) and a non-peak period (N). The costs of handling Product A and Product B are allowed to vary between the peak and non-peak periods. The two products might also have different product-specific costs across the two periods.

The volumes for Product A and Product B are the sums of the non-peak (N) and peak (P) volumes for the two products:

$$V_A = V_{AN} + V_{AP}.$$

$$V_B = V_{BN} + V_{BP}.$$

Rearranging these expressions provides the definitions for the two period-specific volumes for each product:

$$V_{AN} = \theta_{AN} V_A \; ; \; \theta_{AN} = \frac{V_{AN}}{V_{AN} + V_{AP}}.$$

$$V_{AP} = \theta_{AP} V_A \; ; \; \theta_{AP} = \frac{V_{AP}}{V_{AN} + V_{AP}}.$$

$$V_{BN} = \theta_{BN} V_B \; ; \; \theta_{BN} = \frac{V_{BN}}{V_{BN} + V_{BP}}.$$

$$V_{BP} = \theta_{BP} V_B \; ; \; \theta_{BP} = \frac{V_{BP}}{V_{BN} + V_{BP}}.$$

These definitions determine the responses in the time-period volumes to changes in the overall level of volume:

$$\frac{\partial V_{ji}}{\partial V_j} = \theta_{ji}; \quad i = N, P; \quad j = A, B.$$

Total Cost is given by:

$$C = \beta_0 + \beta_{0A} + \beta_{0B} + \beta_{1A} V_{AN} - \beta_{2A} V_{AN}^2 + \beta_{1B} V_{BN} - \beta_{2B} V_{BN}^2 - \beta_3 V_{AN} V_{BN} + \delta_0 + \delta_{0A} + \delta_{0B} + \delta_{1A} V_{AP} - \delta_{2A} V_{AP}^2 + \delta_{1B} V_{BP} - \delta_{2B} V_{BP}^2 - \delta_3 V_{AP} V_{BP}.$$

Marginal Cost for Product A is given by:

$$\frac{\partial C}{\partial V_A} = \beta_{1A} \theta_{AN} - 2\,\beta_{2A} \theta_{AN}^2 V_{AN} - \beta_3 \theta_{AN} V_{BN} + \delta_{1A} \theta_{AP} - 2\delta_{2A} \theta_{AP}^2 V_{AP} - \delta_3 \theta_{AP} V_{BP}.$$

Volume Variable Cost for Product A is given by:

$$VVC_A = \frac{\partial C}{\partial V_A} V_A = \beta_{1A} V_{AN} - 2\,\beta_{2A} V_{AN}^2 - \beta_3 V_{BN} V_{AN} + \delta_{1A} V_{AP} - 2\delta_{2A} V_{AP}^2 - \delta_3 V_{AP} V_{BP}.$$

Marginal Cost for Product B is given by:

$$\frac{\partial C}{\partial V_B} = \beta_{1B} \theta_{BN} - 2\,\beta_{2B} \theta_{BN}^2 V_{BN} - \beta_3 \theta_{BN} V_{AN} + \delta_{1B} \theta_{BP} - 2\delta_{2B} \theta_{BP}^2 V_{BP} - \delta_3 \theta_{BP} V_{AP}.$$

Volume Variable Cost for Product B is given by:

$$VVC_B = \frac{\partial C}{\partial V_B} V_B = \beta_{1B} V_{BN} - 2\,\beta_{2B} V_{BN}^2 - \beta_3 V_{BN} V_{AN} + \delta_{1B} V_{BP} - 2\delta_{2B} V_{BP}^2 - \delta_3 V_{AP} V_{BP}.$$

Total Volume Variable Cost is the sum of the volume variable costs for the two products:

$$VVC = \beta_{1A} V_{AN} - 2\,\beta_{2A} V_{AN}^2 - \beta_3 V_{BN} V_{AN} + \delta_{1A} V_{AP} - 2\delta_{2A} V_{AP}^2 - \delta_3 V_{AP} V_{BP} + \beta_{1B} V_{BN} - 2\,\beta_{2B} V_{BN}^2 - \beta_3 V_{BN} V_{AN} + \delta_{1B} V_{BP} - 2\delta_{2B} V_{BP}^2 - \delta_3 V_{AP} V_{BP}.$$

Non-Volume-Variable Cost is the difference between total cost and total volume variable cost:

$$NVVC = TC - VVC$$

$$= \beta_0 + \beta_{0A} + \beta_{0B} + \beta_{1A}V_{AN} - \beta_{2A}V_{AN}^2 + \beta_{1B}V_{BN} - \beta_{2B}V_{BN}^2 - \beta_3 V_{AN}V_{BN} + \delta_0$$

$$+ \delta_{0A} + \delta_{0B} + \delta_{1A}V_{AP} - \delta_{2A}V_{AP}^2 + \delta_{1B}V_{BP} - \delta_{2B}V_{BP}^2 - \delta_3 V_{AP}V_{BP} - \beta_{1A}V_{AN}$$

$$+ 2\,\beta_{2A}V_{AN}^2 + \beta_3 V_{BN}V_{AN} - \delta_{1A}V_{AP} + 2\delta_{2A}V_{AP}^2 + \delta_3 V_{AP}V_{BP} - \beta_{1B}V_{BN}$$

$$+ 2\,\beta_{2B}V_{BN}^2 + \beta_3 V_{BN}V_{AN} - \delta_{1B}V_{BP} + 2\delta_{2B}V_{BP}^2 + \delta_3 V_{AP}V_{BP}.$$

This simplifies to:

$$NVVC = TC - VVC$$

$$= \beta_0 + \beta_{0A} + \beta_{0B} + \beta_{2A}V_{AN}^2 + \beta_{2B}V_{BN}^2 + \beta_3 V_{BN}V_{AN} + \delta_0 + \delta_{0A} + \delta_{0B} + \delta_{2A}V_{AP}^2$$

$$+ \delta_{2B}V_{BP}^2 + \delta_3 V_{AP}V_{BP}.$$

The positive second order and cross-product terms arise because they appear only once in total cost, but twice in volume variable cost. The total non-volume-variable cost can be decomposed into the non-volume-variable cost for the non-peak period and the non-volume-variable cost for the peak period:

$$NVVC_N = \beta_0 + \beta_{0A} + \beta_{0B} + \beta_{2A}V_{AN}^2 + \beta_{2B}V_{BN}^2 + \beta_3 V_{BN}V_{AN}.$$

$$NVVC_P = \delta_0 + \delta_{0A} + \delta_{0B} + \delta_{2A}V_{AP}^2 + \delta_{2B}V_{BP}^2 + \delta_3 V_{AP}V_{BP}.$$

The incremental cost for a product is the difference between total cost and the cost remaining when the product is removed from the product vector. For example, for product A, incremental cost is:

$$IC_A = TC - TC(-A) = \beta_0 + \beta_{0A} + \beta_{0B} + \beta_{1A}V_{AN} - \beta_{2A}V_{AN}^2 + \beta_{1B}V_{BN} - \beta_{2B}V_{BN}^2 - \beta_3 V_{AN}V_{BN}$$
$$+ \delta_0 + \delta_{0A} + \delta_{0B} + \delta_{1A}V_{AP} - \delta_{2A}V_{AP}^2 + \delta_{1B}V_{BP} - \delta_{2B}V_{BP}^2 - \delta_3 V_{AP}V_{BP} - \beta_0$$
$$- \beta_{0B} - \beta_{1B}V_{BN} + \beta_{2B}V_{BN}^2 - \delta_0 - \delta_{0B} - \delta_{1B}V_{BP} + \delta_{2B}V_{BP}^2.$$

This simplifies to:

$$IC_A = \beta_{0A} + \beta_{1A}V_{AN} - \beta_{2A}V_{AN}^2 - \beta_3 V_{AN}V_{BN} + \delta_{0A} + \delta_{1A}V_{AP} - \delta_{2A}V_{AP}^2 - \delta_3 V_{AP}V_{BP}.$$

Proposal One argues for including all of the non-volume-variable cost for the peak period in the incremental cost for competitive products, but fails to define what else should be included in the proposed incremental cost measure. It is thus incomplete. If the proposal was to add all non-volume-variable cost to the otherwise calculated incremental cost, then it would be invalid because of double counting. Some of the non-volume-variable cost from the peak is already in the incremental cost for competitive products. To avoid this double counting, one could add the entire non-volume-variable cost of the peak to the volume variable cost for competitive products. In the theoretical model that would be the volume variable cost for Product A. In this approach the Proposal One incremental cost for product A is given by:

$$\widetilde{IC}_A = \delta_0 + \delta_{0A} + \delta_{0B} + \delta_{2A}V_{AP}^2 + \delta_{2B}V_{BP}^2 + \delta_3 V_{AP}V_{BP} + \beta_{1A}V_{AN} - 2\,\beta_{2A}V_{AN}^2 - \beta_3 V_{BN}V_{AN}$$
$$+ \delta_{1A}V_{AP} - 2\delta_{2A}V_{AP}^2 - \delta_3 V_{AP}V_{BP.}$$

This simplifies to:

$$\widetilde{IC}_A = \delta_0 + \delta_{0A} + \delta_{0B} + \delta_{1A}V_{AP} - \delta_{2A}V_{AP}^2 + \delta_{2B}V_{BP}^2 + \beta_{1A}V_{AN} - 2\,\beta_{2A}V_{AN}^2 - \beta_3 V_{BN}V_{AN}.$$

This proposed measure for the incremental cost of Product A includes some Product B specific costs and omits the peak-period economies of scope measure.